# Homework 1

1. Can all non-negative real numbers be represented in positional form in any base (base, digits, exponent), i.e.  
   1. Given infinite precision, yes. With finite precision (as in floating point numbers), no.
   2. Integers: any integer can be represented in any base, given a large-enough .
   3. Rationals: any rational number can be represented in *some* base using finite precision, given a choice of . The choice of base depends on prime factorization of the denominator when the number is represented in numerator-denominator form.
   4. Irrationals: irrational numbers cannot be represented in finite precision in *any* base

With infinite precision (the number of digits) but a bounded set of exponents, there are a minimum and a maximum value that can be represented

1. Suppose . What are the range of numbers that can be represented for an arbitrary base, ?
   1. Then we start with

In infinite precision these are the real numbers such that .

1. Characterize the numbers that have an unique representation in an arbitrary base, .
   1. Without restrictions on the representation, no number has a unique representation. For example, the number 1/3 in base 3 could be represented as where ,

i.e “1”, but could also be represented as , i.e “01”.

* 1. A representation that begins with a non-zero digit, , and has a fixed precision, represents a number uniquely. In the previous example, if we take 1 as the leading digit, followed by any number of 0s, then 1/3 can still be represented as “1”e-1 or “10”e-1. While these are arguably different representations, the “e-1” component is uniquely determined. If our representation is required to begin with a non-zero digit ***and*** has a fixed number of digits, then in two digits this can only be represented as “10”e-1.

1. Write a function that takes a decimal number, a base, and the precision, and returns the closest floating point representation, a vector of digits and an exponent, normalized. See “Decimal Representation.html”.
2. Given a base, precision, emin (minimum exponent) and emax (maximum exponent), list all floating point numbers that can be represented. See “List Representable Numbers.html”.